On online Ramsey theory

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Ramsey Theory

Theorem (Ramsey's theorem, 1930)

For positive integers r and s, every graph of sufficiently large order has a complete graph of r vertices or an independent set of s vertices.

Theorem (Van der Waerden's theorem, 1927)

For positive integers c and n, every c-coloring of integers in $\{1, 2, \ldots, N\}$ for sufficiently large N induces a monochromatic arithmetic progression of length n.

Definition of Online Ramsey Game

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An *online Ramsey game* for H on $\mathcal{C}$ is a game between two players, Builder and Painter, with the following rules:

- Each turn:
  - Builder draws finitely many vertices and a new edge so that the resulting graph is in $\mathcal{C}$
  - Painter colors the new edge either red or blue.

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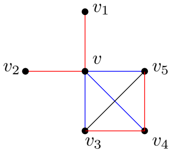
- Each turn:
  - Builder draws finitely many vertices and a new edge so that the resulting graph is in $\mathcal{C}$
  - Painter colors the new edge either red or blue.
- If Builder can force Painter to make a monochromatic copy of H, then Builder wins.
- If Painter can avoid creating a monochromatic copy of H forever, then Painter wins.

Example

- Builder wins the online Ramsey game for C_3 on planar graphs.



Self-unavoidability

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A class $\mathcal{C}$ of graphs is called *self-unavoidable* if Builder wins the online Ramsey game for every $H \in \mathcal{C}$ on $\mathcal{C}$.

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Proposition (Grytczuk, Hałuszczak, and Kierstead, 2004)

Builder wins the online Ramsey game for every k-colorable graph on k-colorable graphs.

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However, this is not true for outerplanar graphs!

On Planar Graphs

Proposition (Grytczuk, Hałuszczak, and Kierstead, 2004)

Builder wins the online Ramsey game for C_n on planar graphs for all n.

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Conjecture (Grytczuk, Hałuszczak, and Kierstead, 2004)
Builder wins the online Ramsey game for H on planar graphs if and only if H is outerplanar.

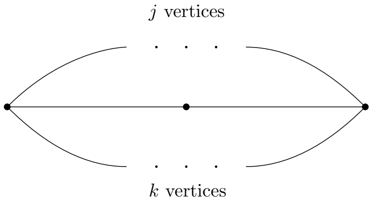
On Planar Graphs

Theorem (Petříčková, 2014)

For every outerplanar graph H, Builder wins the online Ramsey game for H on planar graphs.

Theorem (Petříčková, 2014)

Builder wins the online Ramsey game for $\theta_{2,j,k}$ on planar graphs for even j, k, while $\theta_{2,j,k}$ is not outerplanar.



Forbidden Minor Characterization

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Painter wins the online Ramsey game for C_3 on outerplanar graphs.

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Builder wins the online Ramsey game for C_3 on planar 2-degenerate graphs.

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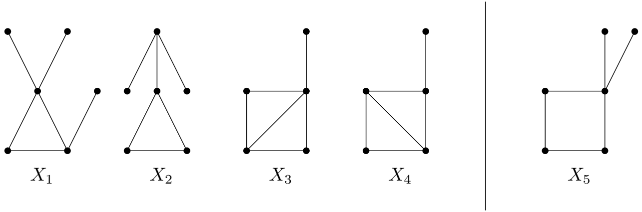
Theorem (C., Choi, Jeong, Oum, 2015+)

Painter wins the online Ramsey game for C_3 on K_4-minor-free graphs.

Forbidden Subgraph Characterization

Theorem (C., Choi, Jeong, Oum, 2015+)

Let F be a connected graph not isomorphic to X_5. Painter wins the online Ramsey game for C_3 on F-free graphs if and only if F is isomorphic to a subgraph of X_i for some $1 \leq i \leq 4$.



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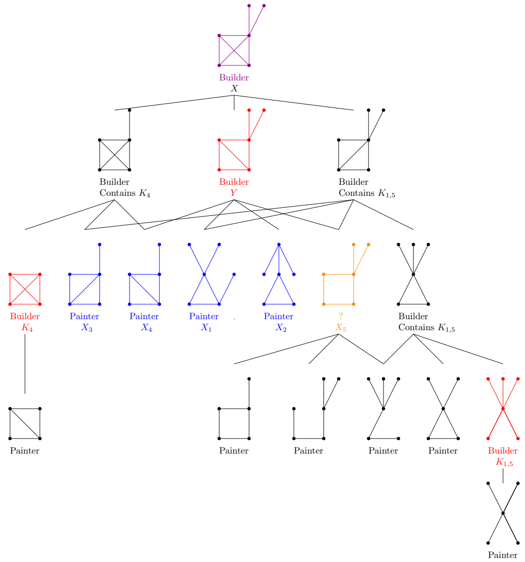
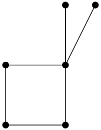


Forbidden Subgraph Characterization

Question

Who wins the online Ramsey game for C_3 on X_5-free graphs?



X_5

Thank you very much.